

# Fixing the number of non-sequential generations within the $SU(2)_L \otimes U(1)_Y$ gauge group

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## Abstract

In this work we explore the possibility to fix the number of new non-sequential chiral-type generations of fermions that could be added to the standard model by combining the condition that arise from the anomalies cancellation with the restriction in the number of flavors impose by the QCD asymptotic freedom. We found that the maximum number of new generations is four, and that allows us at the same time, to place limits for the electrical charges of quarks and leptons within a SM-like framework. Our result is compatible with the constraints involving new generations and their contributions to the oblique parameters  $S$  and  $T$ .

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## I. INTRODUCTION

Although the Standard Model of strong and electroweak interactions [1–3] (SM) is in agreement with all experimental measurements, we have reason to believe that it is a low-energy effective theory of an ultimate fundamental theory of nature, able to answer many of the open questions in the SM, such as the neutrino masses, the hierarchy problem, the number of generations; matter-antimatter asymmetry of the Universe; the strong CP problem and the dark matter content of the Universe among others.

Concerning the number of generations, the SM offers neither an explanation or a justification for why we have three generations of quarks and leptons. Experimentally, we know, from the  $Z$  invisible decay, that the number of light neutrinos is equal to three [4], which implies a compelling proof that there are only three conventional neutrinos with mass below  $M_Z/2 \simeq 45$  GeV, and, by extrapolation, it leads to the idea that there are only three quark-lepton families. But, what is the principle limiting the number of chiral families? Why not have a fourth generation or even more? or why not have leptons or quarks with exotic electric charges?

From the theoretical side, the only upper limit coming from the QCD asymptotic freedom allowing us to include at most sixteen quark flavors or eight quark families [5, 6]. On the other hand, the chiral anomalies cancellation require to have the same number of quark and lepton families, being simple replicas of the first families:  $(\nu_e e^-)_L^T$  and  $(u d)_L^T$ . Although the discovery of the Higgs boson has strongly restricted the possibility of having a sequential fourth family, recent works have shown that by extending the scalar sector, like the Type-II two Higgs doublet model (2HDM) [7] and making use of the so called exact wrong-sign limit where all of the down-type fermions have opposite Higgs coupling to the up-type fermion [8], a fourth family  $(t' b')_L^T$  is still allowed without altering the Higgs production [9]. It is possible because the contributions of the new heavy quarks to the Higgs production via gluon fusion and the respective contributions of the new heavy leptons (necessary for anomalies cancellation) to the  $\gamma\gamma$  and  $Z\gamma$  Higgs decay channels may cancel each other and so, there is no enhancement in Higgs production through gluon-gluon fusion. However, from the heavy Higgs phenomenology, this approach seems to be useful when higher order corrections are considered [10].

Moreover, as it is known, the presence of new fermions can affect the so-called oblique parameters  $S$  and  $T$  which represent the effects of new physics on the  $W$  and  $Z$  vacuum polarization amplitudes (both parameters describe weak-isospin-symmetric and weak isotriplet contributions to  $W$  and  $Z$  loop diagrams, respectively) [11], and whose limits are well established through the electroweak precision data [12]. Thus, by using these parameters, very interesting possibilities have been found regarding the inclusion of new generations of quarks and leptons within or outside the SM framework. In this sense, some authors have found possibilities to have up to two new generations if the neutrino is Dirac-type and up to five new generations if the neutrino is Majorana-type, demanding small differences in mass within the new  $SU(2)$  multiplets [13–16]. If these mass splittings are not small, the compatibility between new generations and the  $S$  and  $T$  parameters is lost. To save this situation we must include new physics contributions, such as new scalar doublets, for example [17].

Otherwise, given the experimental interest of the LHC in the search for quarks and leptons with exotic electrical charges ( $5/3e, -4/3e, -2e$ , etc.), [18] it is possible to include, for example,  $SU(2)_L$  chiral doublets beyond the third generation in a nontrivial way, with different hypercharge from the SM replicas (containing quarks and leptons with exotic electric charges), keeping it free of anomalies and without the need to extend the SM gauge group or go to grand unified theories (GUT) [19–21]. On the other hand, the inclusion of new chiral fermions in higher representations of  $SU(2)_L$  have been studied in the context of the unification of the gauge couplings at a high energy scale [22, 23].

In this letter, we want to show that by including quarks and leptons with exotic charges in the fundamental representation of the SM gauge group, we can fix the number of doublets allowed within the SM, if we combine the cancellation of the anomalies and the asymptotic freedom of the QCD. Additionally, by setting the number of new non-sequential generations, a limit may be placed for exotic electrical charges for both new quarks and leptons.

So, in the next section we will show how to include new quarks and leptons with exotic charges in a safe way, starting from the conventional electric charge operator and analyzing the cancellation of the chiral anomalies arriving to the result of being able to have a limited number of new non sequential generations and allowing us to place constraints to the values of the electrical charges of both leptons and quarks. Finally we will present our conclusions and final remarks.

## II. NEW NON-SEQUENTIAL GENERATIONS IN A SM-LIKE FRAMEWORK

The inclusion of sequential families in the SM is straightforward if we consider the hypercharge values  $\mathcal{Y}_{SM} = 1/6$  (quarks), and  $\mathcal{Y}_{SM} = -1/2$  (leptons) for these new generations. By considering the electric charge operator:

$$Q/e = I_3 + \mathcal{Y} \quad (1)$$

and the chiral anomalies, which have to be canceled in order to guarantee the renormalizability of the theory, we have sequential generations  $(t' b')_L^T$  and  $(\nu e')_L^T$  respectively. Thus, with this choice for  $\mathcal{Y}$  it is clear that we can have  $n$  additional replicas (called sequential generations) to the SM generations.

In principle, the hypercharge assignation for the new particles is arbitrary, but as the simplest assumption, we think that the new quarks and leptons will follow the SM structure, integer electric charge for leptons and fractional electrical charge for quarks. So, as it was already shown[19], the possibility of including quarks with exotic charges ( $5/3 e$ ,  $-4/3 e$ ) within the SM gauge group depends basically on the increase or decrease of the hypercharge by one unit with respect to the SM one. Then, with the new hypercharge  $\mathcal{Y} = \mathcal{Y}_{SM} \pm 1$ , and considering the cancellation of the following anomaly equations:

$$\begin{aligned} [\text{SU}(3)_C]^2 \text{U}(1)_Y &\rightarrow A_C = \sum_Q \mathcal{Y}_{Q_L} - \sum_Q \mathcal{Y}_{Q_R} \\ [\text{SU}(2)_L]^2 \text{U}(1)_Y &\rightarrow A_L = \sum_\ell \mathcal{Y}_{\ell_L} + 3 \sum_Q \mathcal{Y}_{Q_L} \\ [\text{U}(1)_Y]^3 &\rightarrow A_Y = \sum_{\ell, Q} [\mathcal{Y}_{\ell_L}^3 + 3\mathcal{Y}_{Q_L}^3] - \sum_{\ell, Q} [\mathcal{Y}_{\ell_R}^3 + 3\mathcal{Y}_{Q_R}^3] \\ [\text{Grav}]^2 \text{U}(1)_Y &\rightarrow A_G = \sum_{\ell, Q} [\mathcal{Y}_{\ell_L} + 3\mathcal{Y}_{Q_L}] - \sum_{\ell, Q} [\mathcal{Y}_{\ell_R} + 3\mathcal{Y}_{Q_R}], \end{aligned} \quad (2)$$

where  $\mathcal{Y}_{Q_L}$ ,  $\mathcal{Y}_{\ell_L}$ ,  $\mathcal{Y}_{Q_R}$  and  $\mathcal{Y}_{\ell_R}$  are the hypercharges for the  $SU(2)_L$  doublets and singlets for leptons and quarks, we arrive at the following new doublets that include quarks  $X$  and  $Y$  with exotic charges and the necessary inclusion of new lepton doublets containing leptons  $E'$ ,  $E$  and  $F$ , plus the corresponding right-handed singlets. We will refer to these new fermions as exotic quarks and leptons.

$$\begin{aligned}
\psi_L^X &\equiv \begin{bmatrix} X_L \\ U'_L \end{bmatrix} \sim (\mathbf{2}, 7/6), \quad X_R \sim (\mathbf{1}, 5/3), \quad U'_R \sim (\mathbf{1}, 2/3), \\
\psi_L^Y &\equiv \begin{bmatrix} D'_L \\ Y_L \end{bmatrix} \sim (\mathbf{2}, -5/6), \quad D'_R \sim (\mathbf{1}, -1/3), \quad Y_R \sim (\mathbf{1}, -4/3),
\end{aligned} \tag{3}$$

and for the leptons

$$\begin{aligned}
\Psi_L^N &\equiv \begin{bmatrix} E'^+ \\ N_L \end{bmatrix} \sim (\mathbf{2}, 1/2), \quad E'^+ \sim (\mathbf{1}, 1), \quad N_R \sim (\mathbf{1}, 0), \\
\Psi_L^F &\equiv \begin{bmatrix} E^- \\ F_L^{--} \end{bmatrix} \sim (\mathbf{2}, -3/2), \quad E^- \sim (\mathbf{1}, -1), \quad F_R^{--} \sim (\mathbf{1}, -2),
\end{aligned} \tag{4}$$

in which the numbers between parenthesis refers to transformation properties under  $SU(2)_L$  and  $U(1)_Y$ , respectively. The singlets  $E'^+ \sim (\mathbf{1}, 1)$ ,  $E^- \sim (\mathbf{1}, -1)$ , and  $N_R \sim (\mathbf{1}, 0)$  are irrelevant for canceling the anomalies, once the first two form a vector fermion field, and the last has zero hypercharge. As we see, beside the  $X$  quark with electric charge  $5/3$ , we also have the  $Y$  quark with electric charge  $-4/3$ . Thus, the fermion content above extend in  $\pm 1$  the range of the SM particles electric charges allowing for quarks charges  $\mp 4/3, \mp 1/3, \pm 2/3, \pm 5/3$ , and for leptons charges  $0, \pm 1, \pm 2$ .

Now, if we extend our analysis for  $\mathcal{Y}' = \mathcal{Y}_{SM} \pm 2$ , we must include two new generations of quarks containing the additional exotic quarks  $Q$  and  $\mathcal{Q}$  with electric charges  $8/3 e$  and  $-7/3 e$  respectively:

$$\begin{aligned}
\psi_L^Q &\equiv \begin{bmatrix} Q_L \\ X'_L \end{bmatrix} \sim (\mathbf{2}, 13/6), \quad Q_R \sim (\mathbf{1}, 8/3), \quad X'_R \sim (\mathbf{1}, 5/3), \\
\psi_L^{\mathcal{Q}} &\equiv \begin{bmatrix} Y'_L \\ \mathcal{Q}_L \end{bmatrix} \sim (\mathbf{2}, -11/6), \quad Y'_R \sim (\mathbf{1}, -4/3), \quad \mathcal{Q}_R \sim (\mathbf{1}, -7/3),
\end{aligned} \tag{5}$$

and two new leptonic generations with the additional exotic leptons  $\mathcal{E}^{++}$  and  $F^{---}$ :

$$\Psi_L^{\mathcal{E}} \equiv \begin{bmatrix} \mathcal{E}_L^{++} \\ E'^+ \end{bmatrix} \sim (\mathbf{2}, 3/2), \quad \mathcal{E} \sim (\mathbf{1}, 2), \quad E'^+ \sim (\mathbf{1}, 1),$$

$$\Psi_L^{\mathcal{F}} \equiv \begin{bmatrix} F_L^{--} \\ \mathcal{F}_L^{--} \end{bmatrix} \sim (\mathbf{2}, -5/2), \quad F_R^{--} \sim (\mathbf{1}, -2), \quad \mathcal{F}_R^{--} \sim (\mathbf{1}, -3), \quad (6)$$

If we take the new general hypercharge  $\mathcal{Y}' = \mathcal{Y}_{SM} \pm n$ , with  $n = 1, 2, 3, \dots$ , we can include  $n_g = 2n$  non-sequential generations of quarks and leptons, with exotic electric charges expressed in a general form, given by:

$$\begin{aligned} \psi_L^{\mathcal{Q}} &\equiv \begin{bmatrix} \mathcal{Q}_L^{2/3+n_g} \\ \mathcal{X}'_L^{-1/3+n_g} \end{bmatrix} \sim (\mathbf{2}, 1/6 + n_g), \quad \mathcal{Q}_R \sim (\mathbf{1}, 2/3 + n_g), \quad \mathcal{X}'_R \sim (\mathbf{1}, -1/3 + n_g), \\ \psi_L^{\mathcal{K}} &\equiv \begin{bmatrix} \mathcal{Y}'_L^{2/3-n_g} \\ \mathcal{K}_L^{-1/3-n_g} \end{bmatrix} \sim (\mathbf{2}, 1/6 - n_g), \quad \mathcal{Y}'_R \sim (\mathbf{1}, 2/3 - n_g), \quad \mathcal{K}_R \sim (\mathbf{1}, -1/3 - n_g), \end{aligned} \quad (7)$$

and for the leptons

$$\begin{aligned} \Psi_L^{\mathcal{E}} &\equiv \begin{bmatrix} \mathcal{E}^{n_g} \\ E_L^{-1+n_g} \end{bmatrix} \sim (\mathbf{2}, -1/2 + n_g), \quad \mathcal{E} \sim (\mathbf{1}, n_g), \quad E_R \sim (\mathbf{1}, -1 + n_g), \\ \Psi_L^{\mathcal{F}} &\equiv \begin{bmatrix} F_L^{-n_g} \\ \mathcal{F}_L^{-1-n_g} \end{bmatrix} \sim (\mathbf{2}, -1/2 - n_g), \quad F_R \sim (\mathbf{1}, -n_g), \quad \mathcal{F}_R \sim (\mathbf{1}, -1 - n_g), \end{aligned} \quad (8)$$

Therefore, the inclusion of new non-sequential chiral-type doublets with exotic electric charges have to be in pairs, i.e.  $n_g = 2n$  with  $n = 1, 2, 3, \dots$ , in order to maintain the consistency of the model i.e. cancel the gauge anomalies  $A_C=0$ ,  $A_L=0$ ,  $A_Y=0$  and  $A_G=0$ .

Then, with this general result, a natural question arises, how could we fix the maximum value of  $n$ ? It is clear that if the maximum value for  $n$  could be found, in addition to setting the maximum number of allowed generations, we can also limit the maximum value of the electric charge that new leptons and quarks can carry inside this SM-like framework.

So, if we take into account that the asymptotic freedom of the QCD, tested up to  $\vartheta(\text{TeV})$  at the LHC [24], require that the number of flavors of quarks ( $n_F$ ) be equal equal to or less than 16, where 6 flavors are assigned to the SM quarks, then we can still accommodate 10 new flavors of quarks match up in 5 new  $SU(2)$  doublets. With this information and by considering the relation

$$n_g = 2n,$$

which implies that the number of new flavors of exotic quarks for a given value of  $n$  is equal to  $2n_g$ , we get the following relation:

$$2n_g \leq 10 \quad \text{or} \quad n \leq 2.5.$$

Thus, we have found that the maximum number of new non-sequential generations containing quarks and leptons with exotic electrical charges that can be included is four. In addition, with the value of  $n = 2$ , we can limit the allowed electric charges of the new and SM quarks and leptons:

$$\begin{aligned} -\frac{11}{3}|e| &\leq Q_{quarks} \leq \frac{14}{3}|e| \\ -5|e| &\leq Q_{leptons} \leq 4|e| \end{aligned}$$

### III. CONCLUSIONS AND FINAL REMARKS

In this work we have considered the possibility of limiting the maximum number of new generations that can be included within the SM framework, but in one way in which we can have both leptons and quarks with exotic electrical charges. Since the SM does not justify or explain why the existence of only three generations and the absence of fermions with exotic electrical charges, the search for a solution leads to extensions of the gauge group including grand unified theories. However, when considering the possibility of including fermions with exotic electric charges in the SM and maintaining the consistency of the theory through the cancellation of the gauge anomalies, we arrive at a scenario where we can fix the maximum number of new doublets that can be added to the SM spectrum, and at the same time, include exotic electric charges. So, if we consider values of the hypercharge such that the new quarks and leptons accommodated in the new doublets can have exotic electric charges, and maintaining the cancellation already mentioned, and if we take into account that the asymptotic freedom of the QCD also limits the maximum number of flavors of quarks to 16 flavors, we observe that the maximum number of new generations that can be added to the SM is equal to four, being these new generations of the non-sequential type. Now, as it has been shown in the literature, the inclusion of new coloured fermions in different  $SU(3)_c$  representations (triplets, sextets, octuplets and decuplets) affects the running of the  $SU(3)_c$  gauge coupling at high energies, which could in some cases, cause the loss of QCD asymptotic freedom [25, 26]. In that sense, our proposal to include new quarks in the triplet

representation of  $SU(3)$  allows us to have even an asymptotic behavior for the strong gauge coupling. Besides, with this value for  $N_F$ , we can place constraints for the allowed values of the electric charges for quarks and leptons.

On the other hand, if we take into account that the effect of new generations can generate deviations of the oblique parameters  $S$ ,  $T$  very sensitive to new physics, allowing us to limit the number of new generations compatible with the electroweak precision data. As was already shown [17, 19], the introduction of new non-sequential generations (i.e. with  $\mathcal{Y} = \mathcal{Y}_{SM} \pm n$ ) does not affect the oblique parameters  $S$  and  $T$ , if the splitting between the masses of the particles in the doublet is small <sup>1</sup>. Following this reasoning, the proposal of this work to include new doublets is compatible with the deviations allowed for the oblique parameters constrained by the electroweak precision data.

In conclusion, we have shown that by considering the inclusion of new fermions with exotic electrical charges in non-sequential generations and the maximum number of flavors allowed by the QCD, it is possible to indicate a maximum number of new non-sequential generations that can be added to the SM ones. From our analysis we conclude that the maximum number of non-sequential generations allowed is four, having at the end a spectrum of seven generations in total within this SM-like framework, being compatible with the limits imposed by the QCD and by the electroweak precision data.

Finally, we call attention to the fact that the LHC is seeking experimental evidence for fermions with exotic charges and masses above electroweak energy scale through the MoEDAL experiment[27], and so, further phenomenological analysis will deserve our attention in future studies.

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<sup>1</sup> The relation for the  $S$  and  $T$  parameters is show in the appendix



## APPENDIX

In this appendix, we show the general expressions for S and T [17] involving the contributions of new fermions  $(\psi_1, \psi_2)$ , with masses  $(m_1, m_2)$  and whose left-handed components form a doublet  $\Psi \equiv (\psi_{1L} \ \psi_{2L})^T \sim (\mathbf{2}, \mathcal{Y})$  of hypercharge  $\mathcal{Y}$ , and their right-handed components are singlets  $\psi_{1R}, \psi_{2R}$ ,

$$S_\Psi = \frac{N_{C\psi}}{6\pi} \left[ 1 - 2\mathcal{Y} \ln \frac{x_1}{x_2} + \frac{1+8\mathcal{Y}}{20x_1} + \frac{1-8\mathcal{Y}}{20x_2} \right], \quad (9)$$

$$T_\Psi = \frac{N_C}{8\pi s_W^2 c_W^2} F(x_1, x_2), \quad (10)$$

$$U_\Psi = -\frac{N_{C\psi}}{2\pi} \left\{ \frac{x_1+x_2}{2} - \frac{(x_1-x_2)^2}{3} + \left[ \frac{(x_1-x_2)^3}{6} - \frac{1}{2} \frac{x_1^2+x_2^2}{x_1-x_2} \right] \ln \frac{x_1}{x_2} \right. \\ \left. + \frac{x_1-1}{6} f(x_1, x_1) + \frac{x_2-1}{6} f(x_2, x_2) + \left[ \frac{1}{3} - \frac{x_1+x_2}{6} - \frac{(x_1-x_2)^2}{6} \right] f(x_1, x_2) \right\} \quad (11)$$

in which  $N_C = 3$  (1) is the color degree of freedom of quarks (leptons),

$$F(x_1, x_2) = \frac{x_1+x_2}{2} - \frac{x_1 x_2}{x_1-x_2} \ln \frac{x_1}{x_2}$$

$$f(x_1, x_2) = \begin{cases} -2\sqrt{\Delta} \left[ \arctan \frac{x_1-x_2+1}{\sqrt{\Delta}} - \arctan \frac{x_1-x_2-1}{\sqrt{\Delta}} \right] & (\Delta > 0) \\ 0 & (\Delta = 0) \\ \sqrt{-\Delta} \ln \frac{x_1+x_2-1+\sqrt{-\Delta}}{x_1+x_2-1-\sqrt{-\Delta}} & (\Delta < 0) \end{cases}$$

with  $x_i = m_i^2/M_Z^2$ , and  $\Delta = 2(x_1+x_2) - (x_1-x_2)^2 - 1$ ,

It is important to indicate that if we consider new generations with non-SM hypercharges, the calculation show that it is possible to include them without conflicting with the allowed intervals for S and T[19] for a wide range of masses.

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- [1] S.L. Glashow, *Nucl. Phys.*, **22**, 579 (1961).
  - [2] S. Weinberg, *Phys. Rev. Lett.*, **19**, 1264 (1967).
  - [3] A. Salam, in: *Elementary Particle Theory, Nobel Symposium*, 367 (Almqvist and Witsell, Stockholm, 1968).
  - [4] T. Riemann and J. Blumlein, *Proc. of the Zeuthen Workshop on Elementary Particles Nucl. Phys. B (Proc. Suppl.)* **B37** (1994).

- [5] D. Gross and F. Wilczek, *Phys. Rev. Letters*, **26**, 1343 (1973).
- [6] M.D. Politzer, *Phys. Rev. Letters*, **26**, 1346 (1973).
- [7] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher, and J. P. Silva, *Phys. Rep.* **516**, 1 (2012).
- [8] P. M. Ferreira, R. Guedes, M. O. P. Sampaio, and R. Santos, *JHEP* **12**, 067 (2014).
- [9] Dipankar Das *et al.*, *Phys. Rev. D*, **97**,011701(R) (2018).
- [10] Sin Kyu Kang *et al.* *Phys. Rev. D*, **98**,095025 (2018).
- [11] M.E. Peskin and T. Takeuchi, *Phys. Rev. D*, **46**, 381 (1992).
- [12] M. Tanabashi *et al.*, (Particle Data Group), *Phys. Rev. D*, **98**, 030001 (2018).
- [13] V.A. Novikov *et al.*, *Physics Letters B* **529**, 111 (2002).
- [14] Takeo Inami *et al.* *Mod. Phys. Lett. A*, **10**, 1471 (1995).
- [15] M. Maltoni *et al.*, *Physics Letters B* **476**, 107 (2000).
- [16] A. A. Natale and P. S. Rodrigues da Silva *Mod. Phys. Lett. A*, **10**, 1829 (1994).
- [17] Hong-Jian He *et al.*, *Phys. Rev. D*, **64**, 053004 (2001).
- [18] Tobias Golling *Progress in Particle and Nuclear Physics* **90**, 156 (2016).
- [19] Alexandre Alves *et al.*, *JHEP*, **07**, 129 (2013).
- [20] Nicolas Bizot and Michele Frigerios, *JHEP*, **01**, 036 (2016).
- [21] P. H. Frampton *et al.* *Phys. Rep.*, **330**, 263 (2000).
- [22] Lus M. Cebola, D.Emmanuel-Costa, R. Gonzalez Felipe, and C. Simões, *Phys. Rev. D*, **90**, 125037 (2014).
- [23] C. Simões, *Journal of Physics: Conference Series* **631**, 012084 (2015).
- [24] Aaboud, M. *et al.*, *Eur. Phys Jour. C*, **77**, 872 (2017).
- [25] Diego Becciolini *et al.*, *Phys. Rev. D*, **91**, 015010 (2015).
- [26] Javier Llorente, Benjamin P. Nachman, *Nucl. Phys. B*, **936**, 106 (2018).
- [27] B. Acharya *et al.* *Int. J. Mod. Phys. A*, **29**, 1430050 (2014).